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On the calculation of static stability in the 10-level model

by

A. J. Gadd

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1. The thermodynamic equation

In the pressure coordinate system the thermodynamic equation may be written as follows.

$$c_p \frac{DT}{Dt} - \omega \frac{RT}{p} = Q \qquad (1)$$

Q represents the effects of heat sources and sinks ; the other notation is standard.

If potential temperature θ is used as the thermodynamic variable equation (1) becomes the following.

$$c_p \frac{D\theta}{Dt} = \left(\frac{p_{1000}}{p}\right)^{\kappa} Q \qquad (2)$$

where $\theta = T(p_{1000}/p)^{\kappa}$ and $\kappa = R/c_p$

If the atmosphere is assumed to be in hydrostatic equilibrium the geopotential height h may be used as the thermodynamic variable and it may be shown that equation (1) leads to the following.

$$\frac{D}{Dt}\left(\frac{\partial h}{\partial p}\right) + \omega \frac{1-\kappa}{p} \frac{\partial h}{\partial p} = \frac{\kappa}{gp} Q \qquad (3)$$

where, by the hydrostatic relationship, $\partial h/\partial p = -RT/gp$

2. Finite differences

Whereas equations (1) (2) and (3) are, in a hydrostatic atmosphere, exactly equivalent differential equations, the finite difference approximations to them used in a given numerical weather prediction model will not necessary lead to identical results. Since T, θ, and $\partial h/\partial p$ are related to one another by functions of pressure only, horizontal finite differences give exactly equivalent results for all three equations in a pressure coordinate model. In such a model therefore, the alternative forms of the thermodynamic equation can only give differing results through the term in the vertical velocity ω, i.e. through the effects of static stability.

3. Application to the 10 level model

The thermodynamic structure of the 10 level model is defined by the nine thicknesses of the 100mb layers from 100mb to 1000mb. The thick-

ness h' of a layer may be regarded as corresponding to a finite difference approximation to $\partial h/\partial p$.

$$\partial h/\partial p \simeq -h'/\Delta p \qquad \text{where } \Delta p = 100\text{mb}$$

The thickness of a layer as a result of vertical motion according to the equation

$$\partial h'/\partial t = \beta\omega$$

where β is a measure of the static stability.

Using equation (3) and the definition of h' directly the following form is obtained.

$$\frac{\partial h'}{\partial t} = -\left[\frac{\partial h'}{\partial p} + \frac{1-\kappa}{p}h'\right]\omega = \beta_{h'}\omega \qquad \text{say.}$$

Alternatively, the relationship

$$\theta = T\left(\frac{p_{1000}}{p}\right)^{\kappa} = -\frac{gp}{R}\left(\frac{p_{1000}}{p}\right)^{\kappa}\frac{\partial h}{\partial p} = \frac{gp}{R}\left(\frac{p_{1000}}{p}\right)^{\kappa}\frac{h'}{\Delta p}$$

which implies that

$$\frac{\partial h'}{\partial t} = \Delta p\,\frac{R}{gp}\left(\frac{p}{p_{1000}}\right)^{\kappa}\frac{\partial\theta}{\partial t}$$

may be used together with equation (2) to obtain

$$\frac{\partial h'}{\partial t} = \left[-\Delta p\,\frac{R}{gp}\left(\frac{p}{p_{1000}}\right)^{\kappa}\frac{\partial\theta}{\partial p}\right]\omega = \beta_{\theta}\omega \qquad \text{say.}$$

In the 10 level model $\beta_{h'}$ and β_{θ} may be computed using simple centred differences in h' or θ except at 950mb and 150mb. At 950mb one sided differences have been used. At 150mb the use of one sided differences has been avoided by assuming that the atmosphere is isothermal at that level, in which case

$$\left(\frac{\partial h'}{\partial p}\right)_{150} = -\frac{h'_{150}}{p_{150}} \quad ; \quad \left(\frac{\partial\theta}{\partial p}\right)_{150} = -\frac{\kappa\theta_{150}}{p_{150}}.$$

4. Comparison of values calculated for the ICAO standard atmosphere

Using the techniques outlined above $\beta_{h'}$ and β_θ have been calculated from the nine thicknesses of the 100mb layers in the ICAO standard atmosphere. Accurate values of β may be obtained for this atmosphere using the formula

$$\beta_T = \frac{R^2 T}{g^2 p^2}\left(\frac{\partial T}{\partial z} + \frac{g}{c_p}\right)$$

into which the exact ICAO values of T, p, and $\partial T/\partial z$ may be substituted.

p (mb)	$\beta_{h'}$	β_θ	β_T	$\beta_{h'}/\beta_\theta$
150	8.382	8.382	8.263	
250	4.599	2.212	1.061	2.08
350	1.006	0.568	0.577	1.76
450	0.498	0.338	0.366	1.48
550	0.308	0.242	0.255	1.27
650	0.213	0.178	0.188	1.2
750	0.153	0.134	0.145	1.14
850	0.122	0.111	0.116	1.1
950	0.181	0.097	0.095	1.86

It will be seen that β_θ is a significantly better approximation to β_T than is $\beta_{h'}$. A comparatively large error remains at 250mb and is a result of the presence of the ICAO tropopause at 200mb.

5. Results of integrations

Comparative integrations using $\beta_{h'}$ and β_θ as approximations to the static stability term have been computed using the octagon Lax-Wendroff explicit model on 00Z 14.2.73 data and the rectangle semi-implicit and split explicit models on 00Z 2.4.73 data. The results are illustrated in the attached diagrams.

The general impact of the static stability calculation was the same for all three integration schemes. The forecasts using the static stabilities derived from potential temperature gave more rapid development and faster phase speeds in some depressions, greater amplitude in anticyclones, stronger jet streams at 200 and 300mb (examined for octagon forecast only), and greater rates of rainfall (examined in rectangle forecasts only).

Comparison with CFO charts and with observations indicates that the extra development predicted by the models using β_θ results in improved forecasts on these two occasions. There is some suggestion of slightly excessive development: if this proves to be the case in general it may be desirable to reconsider the values assigned to the drag coefficient.

A test was also carried out using a direct finite difference approximation to β_T . This gave results almost identical to the β_θ formulation, but is less efficient from a computational point of view.

6. Note on convective adjustments

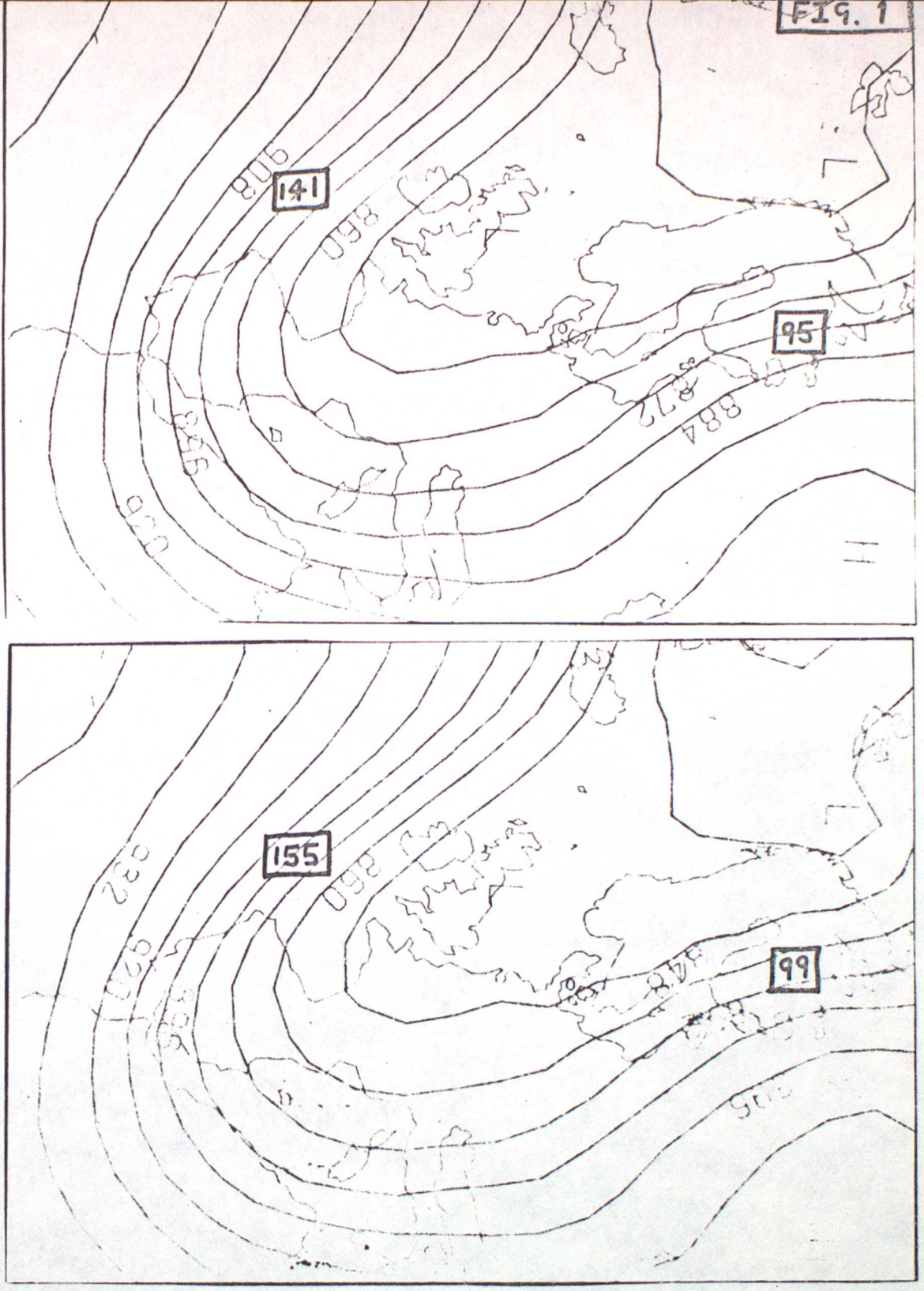
The convective adjustments scheme in the 10 level model is based on the assumption that small-scale convection acts to maintain a neutral lapse rate of temperature $\partial T/\partial z = -\gamma_c$ wherever this would otherwise be exceeded. This neutral lapse rate should be imposed in a manner consistent with the method adopted to calculate the static stability term in the thermodynamic equation. For example, if the static stabilities are derived from potential temperature it is desirable to use the following equation in the convective adjustments scheme.

$$\left(\frac{\partial\theta}{\partial p}\right)_c = -\frac{R\theta}{pg}(\gamma_d - \gamma_c)$$

$\left(\frac{\partial\theta}{\partial p}\right)_c$ represents the neutral vertical gradient of potential temperature; $\gamma_d = g/c_p$ is the dry adiabatic lapse rate.

Use energy argument (Haltiner p140)

FIG. 1



OCTAGON 24 HOUR FORECASTS

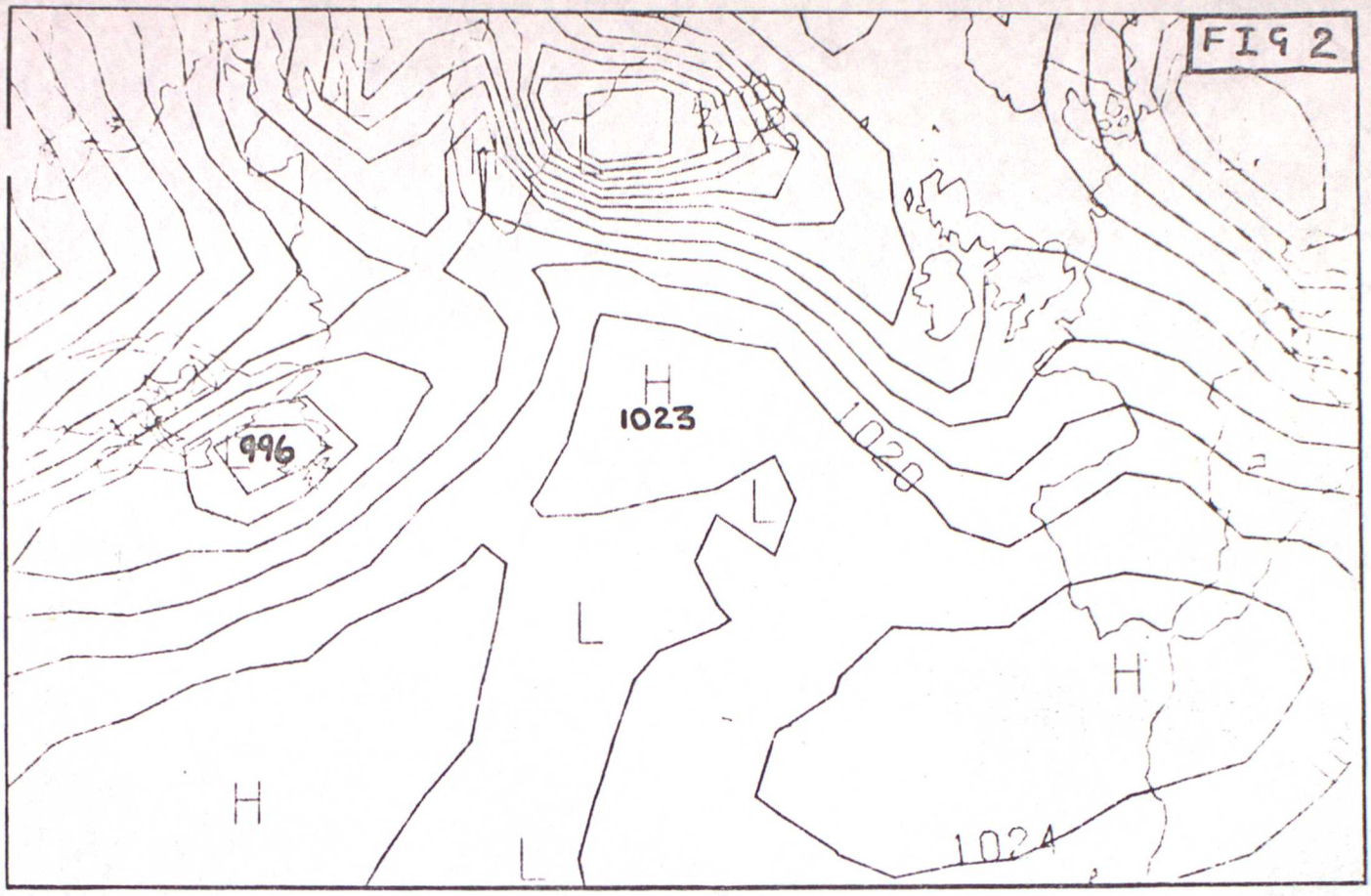
BASED ON 00Z 14.2.73 DATA

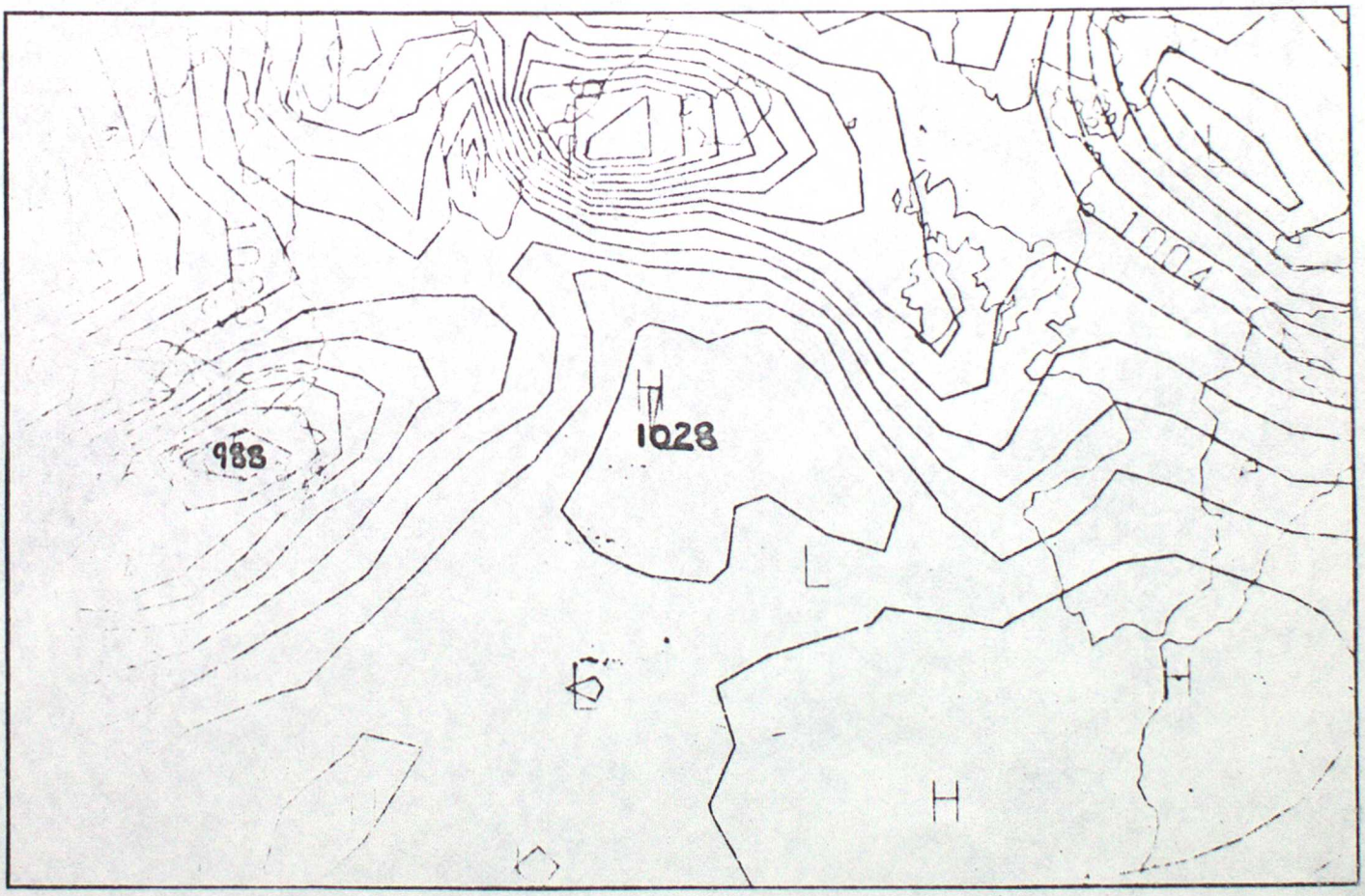
ABOVE : β_h' FORMULATION , BELOW: β_θ FORMULATION

300 MB CONTOURS AND ISOTACH MAXIMA.

The contours are labelled in decameters and the jets are marked in knots within square boxes.

FIG 2



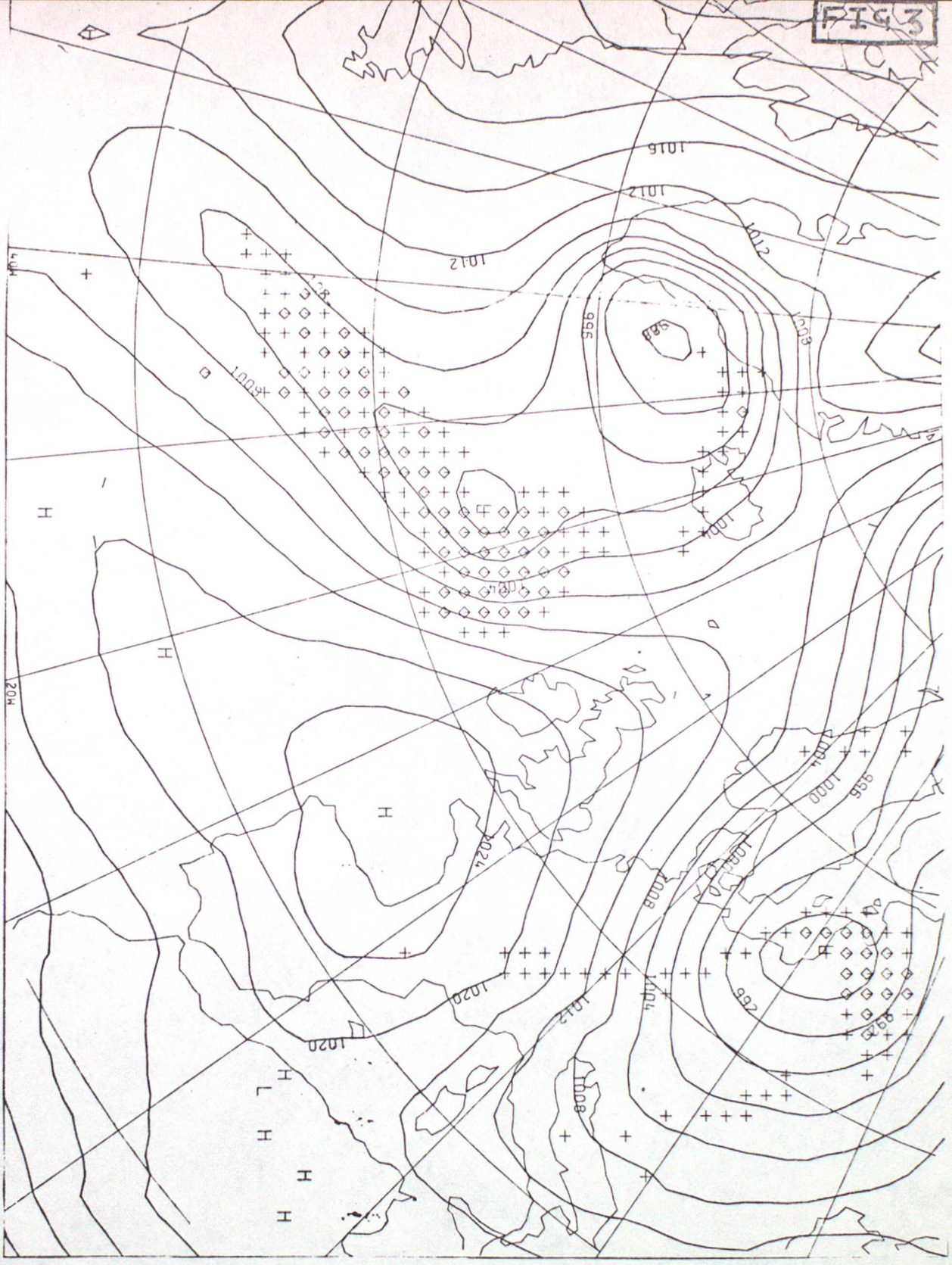


OCTAGON 72 HOUR FORECASTS BASED ON 00Z 14.2.73 DATA

ABOVE : $\beta_{h'}$ FORMULATION , BELOW : β_θ FORMULATION

SURFACE PRESSURE . Both the Newfoundland depression and the North Atlantic anticyclone show increased development and agree more closely with CFO analysis in the β_θ formulation .

FIG 3

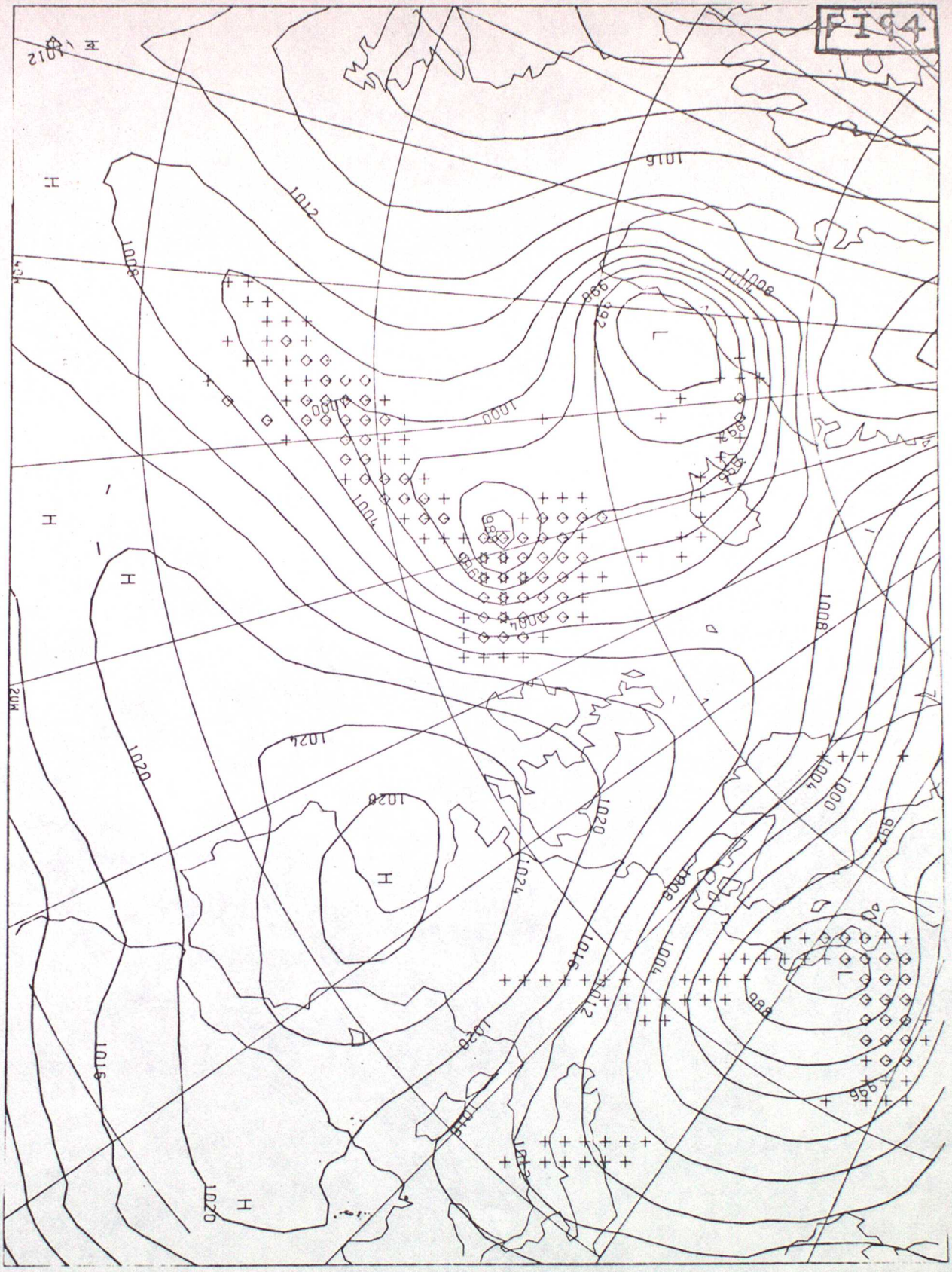


36 Hour Rectangle Forecast : β_h' formulation

Based on 00Z 2.4.73 data

Surface Pressure and Rate of Rainfall

+	0·2 - 0·7	mm/hr
◇	0.7 - 3.0	mm/hr
✡	> 3.0	mm/hr



36 Hour Rectangle Forecast : β_θ formulation

To be compared with previous figure.

The change in the calculation of β has produced increased development and movement of the Atlantic depression and more rapid intensification of the Biscay anticyclone. Note also the increased rates of rainfall.